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Endogenous Market Structure, Occupational Choice, and Growth Cycles

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Abstract

We model an industry that supplies intermediate goods in a growing economy. Agents can choose whether to provide labour or to become firm owners and compete in the industry. The idea that entry is determined through occupational choice has major implications for the economy's intrinsic dynamics. Particularly, the results show that economic dynamics are governed by endogenous volatility in the determination of both the number of industry entrants and in the growth rate of output. Consequently, we argue that occupational choice and the structural characteristics of the endogenous market structure can act as both the impulse source and the propagation mechanism of economic fluctuations.

Keywords: Overlapping generations, Endogenous cycles, Firms' entry, Industry Dynamics

JEL Classification: E32, L16

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1 Introduction

There is an aspect of economic performance that is inherent to both developed and developing countries alike. Specifically, most of them are intrinsically volatile with respect to their economic performance. Of course, the magnitude and duration of economic fluctuations differs among economies. Nevertheless, most economies will experience situations where periods of strong economic activity will be followed by periods of weak increases, or even declines, in measures of economic performance.

Contrary to more conventional approaches that view exogenous (demand and/or supply) shocks as the initial impulse sources behind fluctuations in major economic variables, there is another strand of literature arguing that there is no reason to restrict attention to such exogenous processes as the generating causes of economic volatility.[1] Instead, its impulse source may be embedded in the deep structural characteristics that shape the economy's dynamics and may lead economic variables to display fluctuations, either through damped oscillations or even periodic orbits that are of a more permanent nature. Given that such movements do not rest on the presence of exogenous shocks, they are referred to as 'endogenous volatility' or 'endogenous cycles'. Analyses on this strand of literature include the papers by Grandmont (1985); Benhabib and Nishimura (1985); Reichlin (1986); Azariadis and Smith (1996); Matsuyama (1999); Banerji *et al.* (2004); Dos Santos Ferreira and Lloyd-Braga (2005); and Kaas and Zink (2007) among others. Our paper seeks to contribute to this strand of literature by offering a theory that complements the aforementioned ones in enriching our current understanding on the extent to which endogenous forces can be propagated and manifest themselves in economic cycles.

We are motivated by an emerging literature of research papers that incorporate both endogenous entry and strategic interactions among firms, into fully-fledged dynamic general equilibrium frameworks.[2] The papers by Ghironi and Melitz (2005), Etro and Colciago (2010), Colciago and Etro (2010) and Bilbiie *et al.* (2012) show that such frameworks can outperform real business cycle models in capturing stylised facts of key economic variables

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[1] We refer to analyses that view economic fluctuations as only transitory or short-term phenomena, commonly known as 'business cycles'. The main idea is that various exogenous shocks represent the initial impulse sources whose effect is propagated and manifested in fluctuations of major economic variables. Different strands of literature, such as the real business cycle and the new-Keynesian approaches, have debated on both the impulse sources and the propagation mechanisms that lead to economic fluctuations.

[2] See Etro (2009) and the references therein for a more detailed discussion on this strand of literature.

over the cycle. We also incorporate an endogenous market structure, taking the form of an industry whose firms produce and supply intermediate goods in our dynamic model. Rather than analysing how this structure can propagate the initial impact of an exogenous shock however, we argue that the structural characteristics that determine the equilibrium dynamics of the industry act as both the impulse source and the propagation mechanism that generates fluctuations in output growth. In this respect, our analysis is conceptually closer to the work by Dos Santos Ferreira and Lloyd-Braga (2005) who find that the dynamic equilibrium can converge to endogenous cycles, in overlapping generations (OLG) models with imperfect competition and endogenous entry.

Similarly to these latter analyses, our model makes an explicit distinction between the different stages of an agent's lifetime, made possible by the OLG setting that we employ. The reason why the equilibrium number of competitors in the industry varies over time is different however. In particular, the dynamics of the industry rest on the following structural characteristics. Firstly, the number of agents that choose to become intermediate good producers and join the industry, rather than becoming workers in the final goods sector, is determined through an occupational choice process. In other words, the more familiar zero profit condition is replaced by a condition according to which agents compare the utility associated with a particular choice of occupation. Secondly, contrary to labour, intermediate good production requires some specific training that delays the agent's entrance in the industry for the latter stage of her lifetime.

The combination of these characteristics in an OLG setting introduces rich dynamics with regards to the industry's structure. Particularly, the industry displays *endogenous* volatility; that is, fluctuations in industry entry are not governed by the presence of exogenous shocks. Instead, they are manifested in either damped oscillations or limit cycles. The former occur when the steady state equilibrium is locally stable (a sink); the latter when the conditions for stability are not satisfied (the equilibrium is a saddle point) and the dynamics can display flip – or period doubling – bifurcations. These cyclical trajectories rest on the strong non-monotonicities that pervade the dynamics of the industry. Despite the fact that technological progress is exogenous and firms do not contribute to any productivity-enhancing R&D, these fluctuations generate endogenous cycles in the growth rate of output. These growth cycles are solely associated with the cyclical nature of entry and the corresponding variations in output that result from both the number of intermediate goods and the amount of labour.

Again, growth cycles manifest themselves either through damped oscillations or periodic orbits, depending on the corresponding dynamics for the intermediate goods industry to which we alluded earlier.

All in all, the main message from our analysis can be seen as complementary to existing theories on endogenous market structures within dynamic general equilibirium set-ups and to existing theories on endogenous volatility. With respect to the former, we show that the endogenous determination of industry dynamics is not only a stronger propagation mechanism; it may also represent the actual impulse source of growth cycles. With respect to the latter, we show that the combination of occupational choice and endogenous market structure can represent yet another explanatory factor in the emergence of recurrent cycles in economic activity.

Despite the fact that our endeavour is to present a theoretical framework that offers qualitative implications, rather than quantitative ones, it should be noted that our results are not alien to empirical facts. For example, the data seems to support the idea that business cycles are not just short-lived phenomena. On the contrary, existing work (e.g., Comin and Gertler 2006) has offered evidence showing that cycles are relevant to lower frequencies as well – an outcome that corroborates with our model's OLG structure. Furthermore, there is evidence to suggest the existence of medium- and long-term oscillations in industrial activity (e.g., Geroski 1995; Keklik 2003; Baker and Agapiou 2006) in addition to the more commonly observed short-term movements related to the incidence of business cycles – again, a fact which is in accordance with the main mechanism of our equilibrium results.

The remainder of our analysis is organised as follows. In Section 2 we lay down the basic set up of our economy. Section 3 derives the temporary equilibrium while Section 4 analyses and discusses the dynamic equilibrium and its implications. In Section 5 we conclude.

2 The Economic Environment

Time is discrete and indexed by $t = 0,1,2,...$. We consider an economy composed of a constant population of agents that belong to overlapping generations. Every period, a mass of $n > 1$ agents is born and each of them lives for two periods – youth and old age. During their youth, agents are endowed with a unit of time which they can devote (inelastically) to one of the two available occupational opportunities. One choice is to be employed by perfectly competitive firms who produce the economy's final good. In this case, they receive the competitive salary w_t for their labour services. Alternatively, they can devote their unit of time to some educational activity that will equip them with the ability to use managerial effort and produce units of a specific variety j of an intermediate good when they are old. Intermediate goods are used by the firms that produce and supply the final good. We assume that, once made, occupational choices are irreversible.

The lifetime utility function of an agent born in period by t is given by

$$u_j^t = c_{t,j}^t + \beta[c_{t+1,j}^t - \psi V(e_{t+1,j})], \tag{1}$$

where $\beta \in (0,1)$ is a discount factor, $c_{t,j}^t$ denotes the consumption of final goods during youth, $c_{t+1,j}^t$ denotes the consumption of final goods during old age, $e_{t+1,j}$ is effort and $V(e_{t+1,j})$ is a continuous function that captures the disutility from effort and satisfies $V(0)=0$ and $V'>0$. The parameter ψ is a binomial indicator that takes the value $\psi=0$ if the agent is a worker and $\psi=1$ if the agent is an intermediate good producer. As this notation is important for the clarity of the subsequent analysis, it is important to note that the time superscript indicates the period in which the agent is born whereas the time subscript indicates the period in which an activity actually occurs. The subscript j will be applicable only for producers of intermediate inputs and, thus, will later be removed from variables that are relevant to workers.

We assume that the final good is the numéraire. The production of this good is undertaken by a large mass (normalised to one) of perfectly competitive firms. These firms combine labour from young agents, denoted L_t, and all the available varieties of intermediate goods, each of them denoted $x_{t,j}$, to produce y_t units of output according to

$$y_t = A_t \left[N_t^{-\frac{1}{\theta-1}} \left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^a L_t^{1-a}, \tag{2}$$

where $a \in (0,1)$. The parameter $\theta > 1$ is the elasticity of substitution between different varieties of intermediate goods and N_t gives the number of these different varieties (see

Dixit and Stiglitz 1977).[3] Therefore, the latter variable is the number of entrants operating in the oligopolistic industry at time t. The variable A_t denotes total factor productivity, which we assume to grow at a constant rate $g > 0$ every period. Therefore,

$$A_t = (1+g)^t A_0 , \tag{3}$$

where the initial value $A_0 > 0$ is given. Note that, given the timing of events, the initial period's number of intermediate good firms is also exogenously given by $N_0 \in (1, n)$.

The production of intermediate goods takes place under Bertrand competition among producers. Each of them uses her managerial effort and produces units of an intermediate good according to

$$x_{t,j} = \psi \gamma e_{t,j} , \quad \gamma > 0 , \tag{4}$$

where ψ is the binomial indicator whose role we described earlier. Denoting the price of each intermediate good by $p_{t,j}$, the owner's revenue is given by $p_{t,j} x_{t,j}$. As we indicated above, the cost associated with the managerial activity is the effort/disutility cost characterised by the function $V(\cdot)$.

The process according to which agents choose their occupation involves the comparison of the lifetime utility that corresponds to being either a worker or an intermediate good producer. This problem will be formally solved at a later stage in our analysis. Now we will identify the pattern of optimal consumption choices made by each agent, taking her occupational choice as given.

Suppose that there is a storage technology or, alternatively, a lending opportunity that provides a gross return of $1+r$ $(r \geq 0)$ units of output in period $t+1$ for every unit of output stored/lent in period t.[4] Furthermore, denote the present value of an agent's lifetime income by i_t. Given these, we can write her lifetime budget constraint as

$$c_{t,j}^t + \frac{c_{t+1,j}^t}{1+r} = i_t . \tag{5}$$

Substituting (5) in (1), we can determine $\partial u_j^t / \partial c_{t+1,j}^t$ as follows:

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[3] The scale factor $N_t^{-1/(\theta-1)}$ implies that, in a symmetric equilibrium, $N_t^{-1/(\theta-1)} \left(\sum_{j=1}^{N_t} x_{t,j}^{(\theta-1)/\theta} \right)^{\theta/(\theta-1)} = N_t x_t$.

[4] The constant rate r can be attributed to the idea that we deal with a small open economy.

$$\frac{\partial u_j^t}{\partial c_{t+1,j}^t} = \frac{-1}{1+r} + \beta . \tag{6}$$

As long as $\beta(1+r) < 1$, a condition which we henceforth assume to hold, agents will optimally want to consume all their income during their youth. Given that each worker earns w_t in labour income, we can determine that for those young agents whose choice is to provide labour we have $\psi = 0$, $c_t^{t,\text{worker}} = w_t$ and $c_{t+1}^{t,\text{worker}} = 0$. Therefore, we can use (1) to write the lifetime utility of a worker born in period t as

$$u^{t,\text{worker}} = w_t . \tag{7}$$

For producers, however, the equilibrium characteristics are different. Although they would also prefer to consume during their youth, they can only earn income when old, i.e., when they produce and sell their intermediate products. Furthermore, it is impossible for them to borrow against their future income in order to consume in the first period of their lifetime. The reason for this is twofold. On the one hand, as we established earlier, the young workers alive in period t are not willing to lend any of their income. On the other hand, the old producers alive in period t are also unwilling to lend because they will be dead by the time that repayment of the loan becomes possible. For these reasons, those who decide to be intermediate good producers have $\psi = 1$, $c_{t,j}^{t,\text{producer}} = 0$ and $c_{t+1,j}^{t,\text{producer}} = p_{t+1,j} x_{t+1,j}$. Using these results in (1), we get the lifetime utility of a producer born in period t as

$$u_j^{t,\text{producer}} = \beta[p_{t+1,j} x_{t+1,j} - V(e_{t+1,j})]. \tag{8}$$

With this expression we have completed the basic set up of our economy. In the sections that follow we derive the economy's temporary and dynamic equilibrium, with particular emphasis on the dynamics of the intermediate goods industry.

3 Temporary Equilibrium

For the producers of final goods, profit maximisation implies that each input earns its marginal product. In terms of labour income, we have

$$w_t = (1-a) A_t \left[N_t^{-\frac{1}{\theta-1}} \left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^a L_t^{-a} = (1-a)\frac{y_t}{L_t} . \tag{9}$$

For intermediate goods we have

$$p_{t,j} = A_t L_t^{1-a} a N_t^{-\frac{a}{\theta-1}} \left[\left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^{a-1} \left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}-1} x_{t,j}^{\frac{\theta-1}{\theta}-1} . \tag{10}$$

Multiplying both sides of (10) by $x_{t,j}$ and summing over all j's, we can get

$$\sum_{j=1}^{N_t} p_{t,j} x_{t,j} = aA_t \left[N_t^{-\frac{1}{\theta-1}} \left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^{a} L_t^{1-a} . \tag{11}$$

We can combine Equations (10) and (11) and exercise some straightforward, but tedious, algebra to derive the demand function for an intermediate good. This is given by

$$x_{t,j} = \left(\frac{p_{t,j}}{P_t} \right)^{-\theta} \frac{X_t}{N_t} , \tag{12}$$

where

$$X_t = N_t^{-\frac{1}{\theta-1}} \left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} . \tag{13}$$

Furthermore, using $\sum_{j=1}^{N_t} p_{t,j} x_{t,j} = P_t X_t$, the price index is given by

$$P_t = \left(\frac{1}{N_t} \sum_{j=1}^{N_t} p_{t,j}^{1-\theta} \right)^{\frac{1}{1-\theta}} . \tag{14}$$

The result in (12) is nothing else than the familiar inverse demand function in models with a constant elasticity of substitution between different varieties of goods (Dixit and Stiglitz 1977). In other words, the share of product j in the overall demand for intermediate inputs is inversely related to its relative price. This effect is more pronounced with higher values of θ, i.e., if different varieties are less heterogeneous and, thus, more easily substitutable.

Now let us consider the equilibrium in the labour and final goods markets. With respect to the former, the demand for labour by firms (L_t) must be equal to the supply of labour by young agents. Recall that, in period t, out of the total population mass of n, some agents will decide to set up firms and produce intermediate goods in period $t+1$. The number of these agents is N_{t+1}. Therefore, the labour market equilibrium is

$$L_t = n - N_{t+1}. \tag{15}$$

As for the goods market, let us denote the aggregate demand for final goods by d_t. According to our previous discussion, the demand for final goods is the sum of the demand by young workers and old producers alive in period t, that is $d_t = (n - N_{t+1})c_t^{t,\text{worker}} + \sum_{j=1}^{N_t} c_{t,j}^{t-1,\text{producer}}$. Using previous results, this expression can be written as $d_t = L_t w_t + \sum_{j=1}^{N_t} p_{t,j} x_{t,j}$. However, from the expressions in (2), (9) and (11) it is clear that the unit constant returns technology implies that $y_t = L_t w_t + \sum_{j=1}^{N_t} p_{t,j} x_{t,j}$. Therefore, the final goods market clears since

$$y_t = d_t. \tag{16}$$

Now, we can use $\sum_{j=1}^{N_t} p_{t,j} x_{t,j} = P_t X_t$, $y_t - L_t w_t = \sum_{j=1}^{N_t} p_{t,j} x_{t,j}$, (9), (14) and (16) in (12) to write the demand function for the intermediate good as

$$x_{t,j} = \frac{p_{t,j}^{-\theta}}{\sum_{j=1}^{N_t} p_{t,j}^{1-\theta}} a d_t. \tag{17}$$

The result in (17) indicates the interactions in the pricing decisions made by competing firms. It can be used to solve the utility maximisation problem of an agent who produces intermediate goods. To this purpose, it will be useful to specify a functional form for the effort cost component $V(e_{t+1,j})$. For this reason, and to ensure analytical tractability, we specify

$$V(e_{t+1,j}) = m e_{t+1,j}, \quad m > 0. \tag{18}$$

Writing Equation (17) in terms of period $t+1$ and substituting it together with (4) and (18) in (8), allows us to write the utility function of the producer j as

$$u_j^{t,\text{producer}} = \beta\left(p_{t+1,j} - \frac{m}{\gamma}\right)\frac{p_{t+1,j}^{-\theta}}{\sum_{j=1}^{N_{t+1}} p_{t+1,j}^{1-\theta}} ad_{t+1}\,. \tag{19}$$

Given that firm owners operate under Bertrand competition, their objective is to choose the price of their products in order to maximise their lifetime utility. In other words, their objective is

$$\max_{p_{t+1,j}}\left\{\beta\left(p_{t+1,j} - \frac{m}{\gamma}\right)\frac{p_{t+1,j}^{-\theta}}{\sum_{j=1}^{N_{t+1}} p_{t+1,j}^{1-\theta}} ad_{t+1}\right\}. \tag{20}$$

After some straightforward algebra, it can be shown that the solution to this problem leads to a symmetric equilibrium for which

$$p_{t+1,j} = p_{t+1} \text{ and } x_{t+1,j} = x_{t+1} \;\; \forall j\,, \tag{21}$$

where the optimal price equals

$$p_{t+1} = \frac{m}{\gamma}\frac{[\theta(N_{t+1}-1)+1]}{(\theta-1)(N_{t+1}-1)}. \tag{22}$$

In addition, given (17) and (22), the equilibrium quantity of the intermediate good by each entrepreneur is

$$x_{t+1} = \frac{ad_{t+1}}{N_{t+1}}\frac{\gamma}{m}\frac{(\theta-1)(N_{t+1}-1)}{[\theta(N_{t+1}-1)+1]}. \tag{23}$$

The result in Equation (22) resembles the familiar condition according to which the price is set as a markup over the marginal cost of production. In this case, each producer sets a markup over the marginal utility cost of producing the intermediate good, since one unit of production requires a utility cost of m/γ units of effort. Naturally, the markup is decreasing in the number of producers because the latter implies a more intensely competitive environment. Additionally, the markup is also decreasing in θ because higher values of this parameter increase the degree of substitutability between different varieties of intermediate goods – yet another structural characteristic that enhances the degree of competition. From (23), we can see that the inverse demand function implies that the components that reduce the relative price of the input increase its share on aggregate demand.

The solutions above allow us to rewrite the utility of an intermediate good producer, after substituting (16), (21) and (22) in (19), as follows:

$$u^{t,\text{producer}} = \frac{\beta a y_{t+1}}{\theta(N_{t+1}-1)+1}. \tag{24}$$

With this result at hand, we can now turn our attention to the occupational choice problem of an agent who is young in period t.

3.1 Occupational Choice

Our purpose in this section is to determine how many agents will decide to become suppliers of intermediate inputs. Obviously, the equilibrium condition requires that an agent born in t should be indifferent between the two different occupational opportunities. Formally, a condition that needs to hold in equilibrium is

$$u^{t,\text{producer}} = u^{t,\text{worker}}, \tag{25}$$

or, after utilising (7), (9) and (24),

$$\frac{\beta a y_{t+1}}{\theta(N_{t+1}-1)+1} = (1-a)\frac{y_t}{L_t}. \tag{26}$$

We can manipulate algebraically the expression in (26) even further. First, we can use the symmetry condition (Eq. 21) in (2) to get

$$y_t = A_t (N_t x_t)^a L_t^{1-a}. \tag{27}$$

Next, we substitute (16) in (23) to get

$$N_{t+1}x_{t+1} = a y_{t+1}\frac{\gamma}{m}\frac{(\theta-1)(N_{t+1}-1)}{[\theta(N_{t+1}-1)+1]} \Leftrightarrow N_t x_t = a y_t \frac{\gamma}{m}\frac{(\theta-1)(N_t-1)}{[\theta(N_t-1)+1]}. \tag{28}$$

Further substitution of (28) in (27) allows us to write

$$y_t = A_t^{\frac{1}{1-a}}\left\{\frac{a\gamma}{m}\frac{(\theta-1)(N_t-1)}{[\theta(N_t-1)+1]}\right\}^{\frac{a}{1-a}} L_t. \tag{29}$$

Finally, we can use (15), (28) and (29) in (26), and rearrange to get

$$\frac{n-N_{t+2}^E}{\theta(N_{t+1}-1)+1} = \frac{(1-a)}{a\beta(1+g)^{1/(1-a)}}\left\{\frac{[\theta(N_{t+1}-1)+1]}{(N_{t+1}-1)}\frac{(N_t-1)}{[\theta(N_t-1)+1]}\right\}^{a/(1-a)}, \tag{30}$$

where $1+g=\dfrac{A_{t+1}}{A_t}$ is derived by alluding to (3). Note that the superscript in N^E_{t+2} denotes the expectation formed on this variable.

The result in Equation (30) is the most important in our set-up. It implies that the determination of the equilibrium number of firms in the intermediate goods industry is not a static one. Instead, there will be some transitional dynamics as the number of producers converges to its long-run equilibrium. Particularly, we can see that the equilibrium number of firms in any given period depends on both the predetermined number of firms from the previous period and the expectation on the number of firms that will be active during the next period. Note that the endogenous occupational choice is critical for these dynamics. It is exactly because of this choice that the determination of N_{t+1} is related to the previous period's demand conditions (and, thus, N_t) and the next period's labour market equilibrium (therefore N^E_{t+2}).

The intuition for these effects is as follows. If the existing number of intermediate good firms is large, then the overall amount of intermediate goods and, therefore, the marginal product of labour will be higher. This increases the equilibrium wage and thus, the relative benefit from the utility of being a worker when young, rather than setting up a firm when old. Now suppose that, while forming their occupational choice, the current young expect that the future number of firms in the intermediate goods industry will be high. For them, this implies that the amount of labour and, therefore, total demand in the next period will be relatively low. Thus, the relative utility benefit of being a firm owner when old, rather than a worker when young, is reduced because the expectation of lower future demand for final goods will have corresponding repercussions in terms of reduced future demand for intermediate goods as well. Consequently, a reduced number of individuals, out of the current young, will opt for the choice of becoming intermediate good producers.

4 Dynamic Equilibrium

The remainder of our analysis will focus on the dynamics of the industry that produces intermediate goods. In what follows, we consider equilibrium trajectories that satisfy $N^E_{t+2}=N_{t+2}$.

4.1 The Steady State

We can obtain the stationary equilibrium for the number of intermediate good firms, after substituting $N_{t+2}^{E} = N_{t+2}$ in (30) and using the steady state condition $N_{t+2} = N_{t+1} = N_t = \hat{N}$. This procedure will eventually allow us to derive

Proposition 1. *Suppose* $n > 1+\delta$ *where* $\delta = (1-a)/a\beta(1+g)^{1/(1-a)}$. *Then there exists a unique steady state equilibrium* $\hat{N} \in (1,n)$ *such that*

$$\hat{N} = \frac{n + (\theta-1)(1-a)/a\beta(1+g)^{1/(1-a)}}{1+\theta(1-a)/a\beta(1+g)^{1/(1-a)}}. \tag{31}$$

As long as the steady state solution is asymptotically stable, then for any predetermined $N_0 \in (1,n)$ the equilibrium number of producers will eventually converge to $\hat{N}$ in the long-run. Later, we are going to formally characterise the conditions for the (local) stability of this equilibrium. For now, it is instructive to undertake some comparative statics to identify the effects of the economy's structural parameters on the steady state number of firms competing in the intermediate goods industry. This is a task that can be easily undertaken through the use of Equation (31). The results can be summarised in

Proposition 2. *The long-run equilibrium number of firms in the intermediate goods industry is:*

i. *Increasing in the growth rate of total factor productivity* (g) *and the relative weight attached to old age consumption (the discount component* β*);*
ii. *Decreasing in the relative share of labour income* $(1-a)$ *and the degree of substitutability between different varieties of intermediate products* (θ).

The economic interpretation for these results is as follows. A permanent increase in the growth rate causes future demand to become even higher compared to current demand because of the increase in the economy's resources. This effect boosts the relative utility benefit of becoming a firm owner, with corresponding implications for the occupational choices made by young agents. An increase in the relative share of labour income will motivate more agents to work for final goods firms, as the income earned from activities in the intermediate goods industry becomes relatively low. The utility benefit of suchactivities is also impeded in an industry where goods are less heterogeneous. Finally, when individuals discount the utility from old age consumption less heavily, then they have a greater incentive to opt for the occupation from which old age consumption accrues – in this case, firm ownership.

4.2 Transitional Dynamics

Let us use $N_{t+2}^{E} = N_{t+2}$ in (30) and solve the resulting expression for N_{t+2}. Eventually, we get

$$N_{t+2} = n - \frac{(1-a)}{a\beta(1+g)^{1/(1-a)}} \frac{[\theta(N_{t+1}-1)+1]^{1/(1-a)}}{(N_{t+1}-1)^{a/(1-a)}} \left[\frac{N_t - 1}{\theta(N_t - 1)+1} \right]^{a/(1-a)} = F(N_{t+1}, N_t). \quad (32)$$

As we can see, the dynamics of the intermediate goods industry are characterised by a non-linear, second-order difference equation in terms of the industry's size (i.e., the number of agents who compete in the industry).

One way to analyse the transition equation in (32) is to define $Z_t = N_{t+1}$ and treat the dynamics as being generated by the following system of first-order difference equations:

$$Z_{t+1} = F(Z_t, N_t) = n - \delta \frac{[\theta(Z_t - 1)+1]^{1/(1-a)}}{(Z_t - 1)^{a/(1-a)}} \left[\frac{N_t - 1}{\theta(N_t - 1)+1} \right]^{a/(1-a)}, \quad (33)$$

$$N_{t+1} = H(Z_t, N_t) = Z_t, \quad (34)$$

where $N_0, Z_0 \in (1, n)$ are taken as the initial conditions and the steady state satisfies $\hat{Z} = \hat{N}$ and δ is defined in Proposition 1. The Jacobian matrix associated with the planar system of Equations (33)-(34) is

$$\begin{pmatrix} F_{Z_t}(\hat{Z}, \hat{N}) & F_{N_t}(\hat{Z}, \hat{N}) \\ H_{Z_t}(\hat{Z}, \hat{N}) & H_{N_t}(\hat{Z}, \hat{N}) \end{pmatrix},$$

where $\hat{N} = \hat{Z}$ is given in (31). Furthermore, the eigenvalues are the roots of the polynomial $\lambda^2 - T\lambda + D$, i.e.,

$$\lambda_1 = \frac{T - \sqrt{T^2 - 4D}}{2} \text{ and } \lambda_2 = \frac{T + \sqrt{T^2 - 4D}}{2},$$

where

$$T = F_{Z_t}(\hat{Z},\hat{N}) + H_{N_t}(\hat{Z},\hat{N}),$$

and

$$D = F_{Z_t}(\hat{Z},\hat{N})H_{N_t}(\hat{Z},\hat{N}) - F_{N_t}(\hat{Z},\hat{N})H_{Z_t}(\hat{Z},\hat{N}),$$

are respectively the trace and the determinant of the matrix. As it is well known (Azariadis 1993; Galor 2007) the eigenvalues can be used to check the stability of the steady state solution and to trace the transitional dynamics towards it. Later, it will transpire that, under different conditions, $\hat{N}$ can be either (locally) stable or unstable. For now, we will focus our attention to a case whereby the steady state equilibrium characterised by (31) is actually stable, while the possible implications that arise in the scenario where $\hat{N}$ is unstable will be discussed subsequently.

Let us begin by defining $\Xi(\delta) \equiv \delta + \frac{\delta a(1+\delta\theta)}{1-a}\frac{2}{\delta\theta - 1}$ and $\tilde{\delta}$ such that $\Xi(\tilde{\delta}) = n-1$. Furthermore, the analysis that follows will be making use of the following assumptions:

Assumption 1. $n > 1 + 2\left(1 + \frac{3a}{1-a}\right)$,

Assumption 2. $\delta\theta \le 2$.

Both assumptions are employed to make the analysis of the transitional dynamics more precise, clear and sharply focused. Assumption 1 is sufficient to guarantee that the eigenvalues of the dynamical system in (33)-(34) are real. In addition, it allows us to pinpoint a clear parameter condition under which a flip bifurcation may occur. Assumption 2 is sufficient to guarantee that, at least for some range of parameter values, the steady state equilibrium will be stable. The proofs to the subsequent results are relegated to the Appendix.

The sufficient conditions for stability are formally described in

Lemma 1. *If either (i)* $\delta\theta \leq 1$ *or (ii)* $\delta\theta \in (1,2]$ *and* $\delta < \tilde{\delta}$ *, then the steady state solution* $\hat{N}$ *is locally stable. In other words, the dynamics starting from an initial value* $N_0 \in (1,n)$ *will eventually converge to* $\hat{N}$.

With respect to output, once the industry converges to its steady state, the production of final goods will converge to a balanced growth path. Along this path, output will grow at a constant rate that is proportional to the growth rate of total factor productivity. It is straightforward to use Equations (15) and (29) and the result of Lemma 1 to establish that

$$\lim_{t\to\infty}\left(\frac{y_{t+1}}{y_t}-1\right)=(1+g)^{\frac{1}{1-a}}-1. \tag{35}$$

Nevertheless, during the transition to the balanced growth path, the dynamics of output will also be (partially) dictated by the transitional dynamics of the intermediate goods industry. A technical condition that can facilitate a better understanding of how the intermediate goods industry evolves over time is given by

Lemma 2. *As long as the conditions for stability that are summarised in Lemma 1 hold, both eigenvalues are negative , i.e.,* $\lambda_1, \lambda_2 \in (-1,0)$.

Using Lemma 2 we can characterise the transitional behaviour of the economy through

Proposition 3. *Given Lemma 2, the number of firms in the intermediate goods industry converges to its long-term equilibrium through cycles. Consequently, output growth displays fluctuations as it converges to the balanced growth path.*

Recall that the number of producers in any given period is affected by both the predetermined number of producers from the previous period and the expectation on the number of producers that will be active in the future. The manner and direction of these effects, both discussed at an earlier point of our analysis, render the result of Proposition 3 to be a quite intuitive one. For example, consider a situation where the existing number of intermediate good producers is low relative to the steady state. For the current young agents, the incentive to opt for industry entry when old is enhanced because the marginal product of labour (and, therefore, the wage) is currently low. As a result, an increased fraction of the current young will choose to become firm owners and compete in the intermediate goods industry when they become old. However, for this to happen they also need to expect that, next period, a lower fraction of the future generation's agents will decide to become producers because this will increase labour and, therefore, aggregate demand during the period where producers will reap the benefits of their activity. The mechanism that we described previously does verify this expectation, hence granting an even greater incentive to the agents for setting up intermediate good firms. Furthermore, it explains why the size of the intermediate goods industry, and output growth, converge to their long-run equilibrium through cycles.

For illustrative purposes, in what follows we will analyse the transition equation in (32) numerically, making sure to choose parameter values that render the solution in (31) stable, hence a meaningful one. We should emphasise, however, that we undertake these numerical simulations solely as a means to illustrate the transitional behaviour of the economy. The focus of our analysis is still purely qualitative, hence it is neither our intention nor do we claim any attempt to offer a quantitative match of key moments from stylised facts.

For the baseline parameter values, we choose $a = 0.5$, $g = 0.15$, $\beta = 0.95$ and $\theta = 1.25$, while the total population is set to $n = 100$.[5] The initial values are $N_0 = 20$ and $N_1 = 75$ – recalling that N_1 corresponds to Z_0 in (34). In Figure 1 we see the transitional dynamics for the intermediate goods industry, based on this simulation. Given the numerical example, the steady state value for $\hat{N}$ is roughly 50 and the industry converges to this number. Nevertheless, this convergence is clearly non-monotonic. Instead, convergence takes place through damped oscillations, or cycles, during which the number of firms takes values above and below the stationary value as the industry approaches towards it. In Figures 2 we use the same baseline parameter values, in addition to $A_0 = 10$, to simulate the movements of the growth rate of output, $\frac{y_{t+1}}{y_t} - 1$. Again, we can see that, due to fluctuations that occupational choice generates in the determination of the number of producers each period, output converges to its balanced growth path through cycles. Note however that these fluctuations

[5] It can be easily established that these parameter values lie on the permissible range that guarantees stability according to Lemma 1.

are not due to the fact that the intermediate goods industry is associated with some type of R&D that increases the rate of technological progress endogenously. Instead, they are purely associated with variations in output that result from the cyclical nature of N_t and the corresponding variations in both the number of intermediate goods and the labour input.

Before we proceed to the analysis of limit cycles, we will discuss the possibility of indeterminacy in the transitional dynamics of the economy. As we have seen from the second order transition equation in (32), or the equivalent dynamical system in (33)-(34), the transitional dynamics are traced after we consider two initial values N_0 and Z_0 – the latter corresponding to N_1. Nevertheless, while N_0 is indeed predetermined, this is not the case for N_1. Instead, taking the value of N_0 as given, N_1 reflects an equilibrium formed on an expectation about N_2 and so on. In other words, the stability of the steady state equilibrium $\hat{N}$ implies that, for the same $N_0 \in (1, n)$, there are certainly more than one trajectories that are consistent with the economy's convergence to the steady state. In other words, economies that are identical both in terms of structural parameters and predetermined conditions may display very different equilibrium characteristics for a large part of their transition towards the common steady state.[6]

[6] In this respect, our result echoes the main implications of the analysis by Mino *et al.* (2005). They also use an overlapping generations setting to show that occupational choice can be responsible for dynamic indeterminacy. However, there are notable differences between their setting and ours. Firstly, they do not endogenise the number of firms that operate in a particular sector; instead, they assume that both sectors in the economy (producing consumption and investment goods) are perfectly competitive. Secondly, the occupational choice entails a decision on whether to become a skilled worker or remain unskilled – with both types of labour being imperfect substitutes in production. Therefore, the aim and implications of our paper differ significantly.

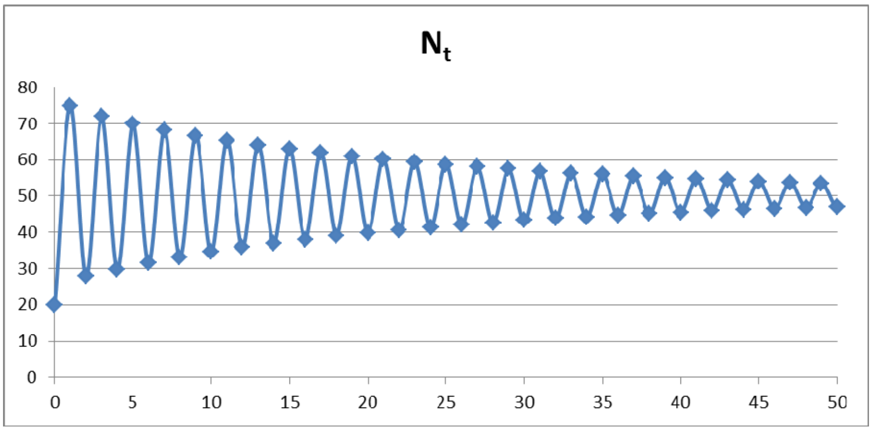

Figure 1. Industry dynamics in the baseline case

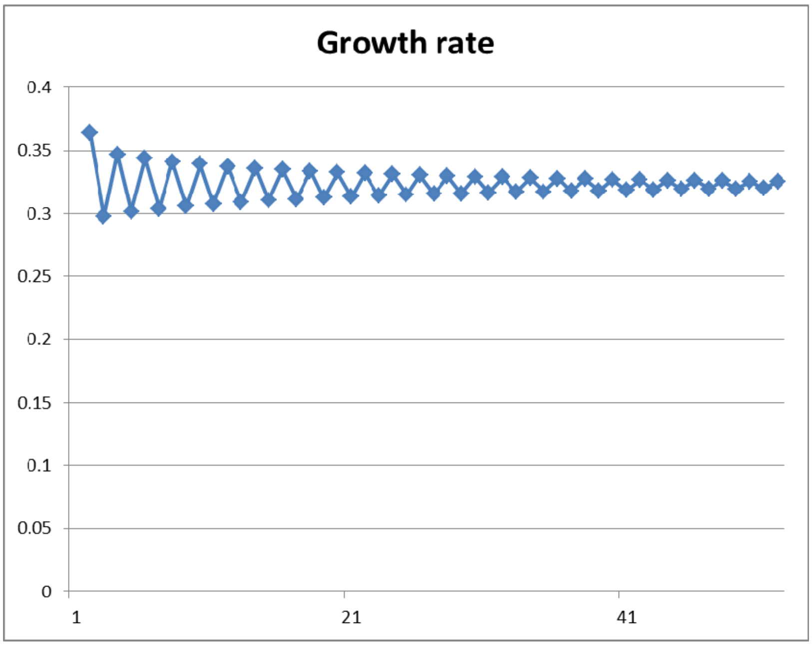

Figure 2. Output growth in the baseline case

4.3 Periodic Equilibrium

So far, we have seen scenarios in which oscillations in economic variables are not permanent – an outcome related to our restriction on conditions that guarantee the stability of the steady state. Nevertheless, it will also be interesting to examine the possibilities that arise when the steady state in (31) does not satisfy these stability conditions. This may happen in circumstances that are described in

Lemma 3. *Suppose that* $\delta\theta \in (1,2]$ *and* $\delta > \tilde{\delta}$. *In this case, the two eigenvalues* λ_1 *and* λ_2 *satisfy* $0 > \lambda_2 > -1 > \lambda_1$. *Therefore, the steady state solution* $\hat{N}$ *is a saddle point.*

The saddle point property of the steady state implies that, for given N_0, there is only one corresponding $N_1(=Z_0)$ such that the industry dynamics follow a path of convergence towards $\hat{N}(=\hat{Z})$ and output converges towards the balanced growth path. All other paths will diverge away from this point. Now, recall that the dynamics are traced after we consider two initial values N_0 and $N_1 = (Z_0)$ from which only N_0 is predetermined. This implies that we can rule out some constantly divergent paths because they are clearly not optimal: as N_t will at some point approach either 1 or n, output and consumption will become equal to zero. Nevertheless, there are paths that although not converging towards $\hat{N}$, there is no reason why they should be ruled out. These paths entail the presence of a periodic equilibrium or limit cycles. We will use the previous numerical example to illustrate such cases, bearing in mind that parameter values must satisfy the conditions summarised in Lemma 3.

In the baseline numerical example, we set the discount factor equal to $\beta = 0.9$. Doing so, the simulation indicates that the number of firms converges to a period-2 cycle equal to $\{N^1, N^2\} = \{76, 22\}$ which corresponds to a period-2 cycle for the growth rate $\{0.358, 0.287\}$ (see Figures 3-4). A period-2 cycle appears as we reduce β even further, until at some point we observe that it becomes unstable and replaced by the emergence of a stable period-4 cycle. For example, setting $\beta = 0.81$ leads to $\{N^1, N^2, N^3, N^4\} = \{93, 8, 90, 5\}$ and a corresponding period-4 cycle for the growth rate $\{0.565, 0.202, 0.454, 0.118\}$ (see Figures 5-6).[7] Reducing β even more leads to the emergence of cycles of period-6, period-8 and so on.

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[7] Note that when writing the periodic equilibria for the number of intermediate good producers, we approximate by using the closest integer.

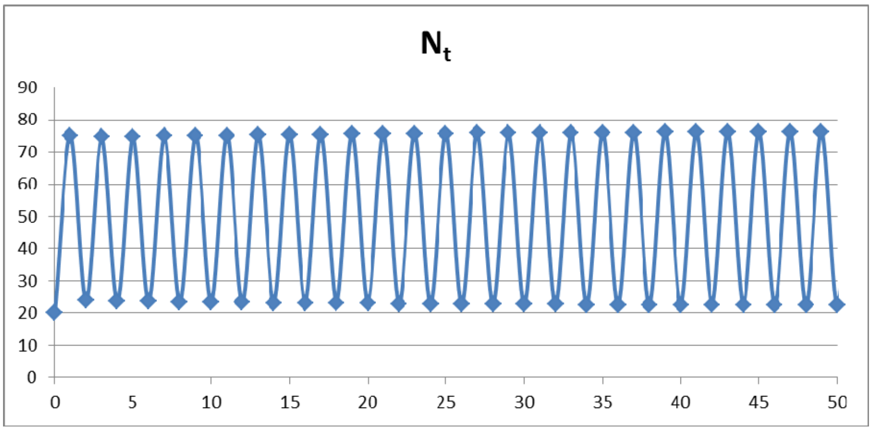

Figure 3. Period-2 cycle for N_t

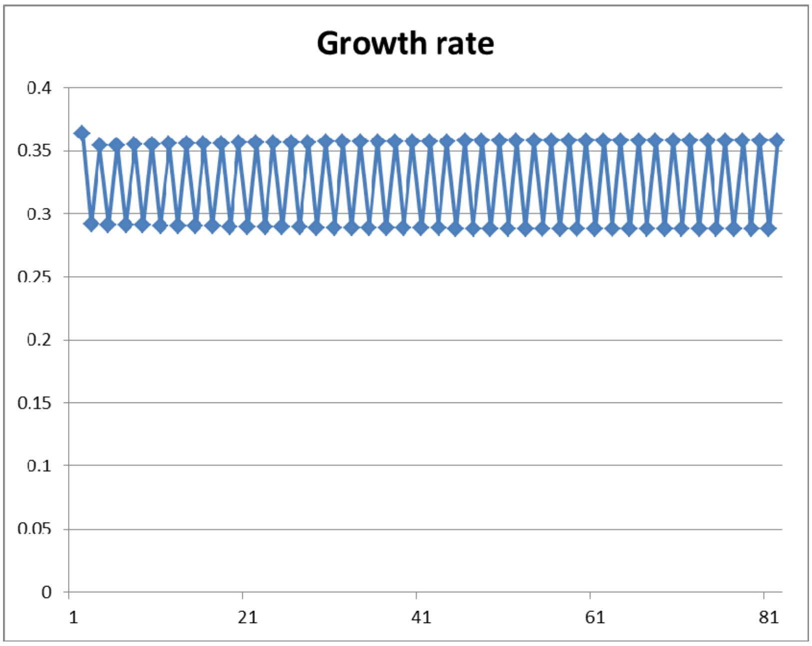

Figure 4. Period-2 growth cycle

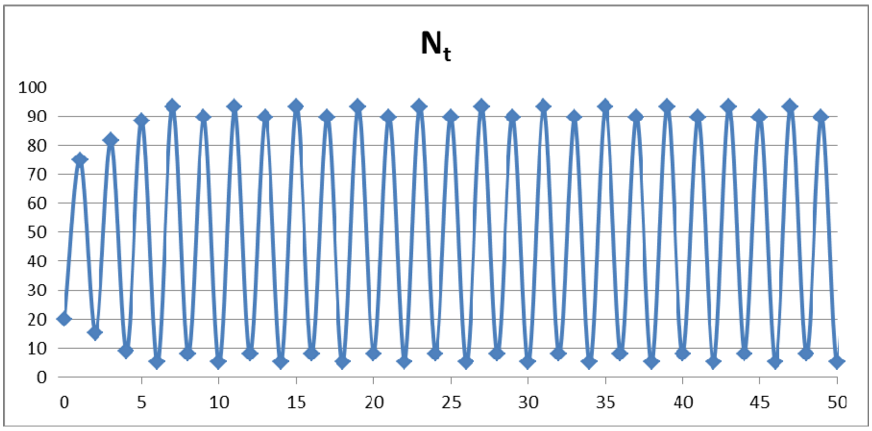

Figure 5. Period-4 cycle for N_t

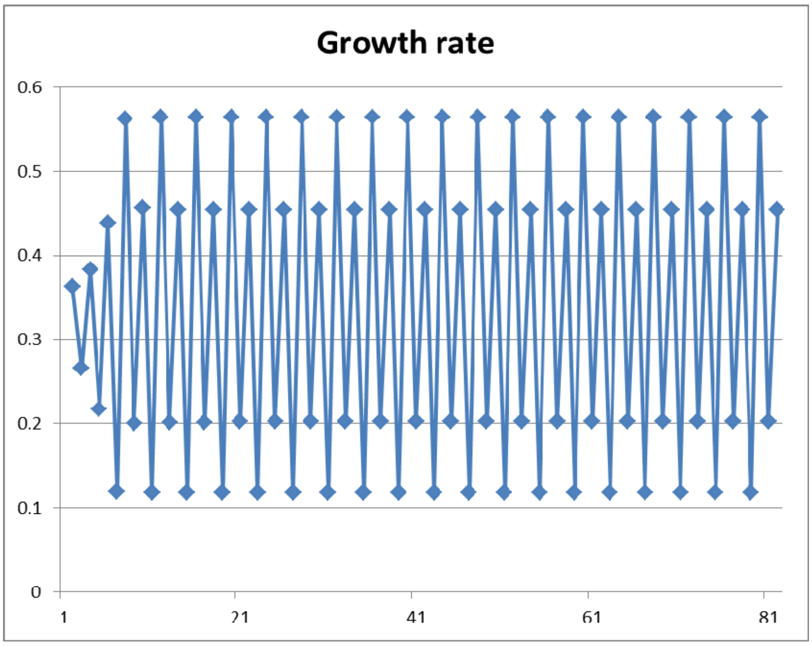

Figure 6. Period-4 growth cycle

It is possible to generalise the implications offered by these numerical examples. We can start with

Lemma 4. *Suppose that* $\delta\theta \in (1,2]$. *The dynamical system of (33) and (34) undergoes a flip (period doubling) bifurcation at* $\delta = \tilde{\delta}$. *Hence, there exist stable limit cycles of at least two periods.*

Given this, we can characterise the dynamics in this case through

Proposition 4. *Under the conditions in Lemmas 3 and 4, fluctuations in the number of intermediate good firms can become permanent. Therefore, output may not converge to its balanced growth path; instead it will fluctuate permanently around it.*

Recall that δ is a composite parameter term that is negatively related to β. Given Lemma 4, it is not difficult to understand why our previous simulations revealed that reductions of the discount factor generate period doubling bifurcations. In terms of intuition, we can allude to the forces of industry dynamics that we described previously. Now however, the impact of non-monotonicities is strong enough so that cycles do not dissipate over time. On the contrary, they become a permanent characteristic of the industry's dynamics and consequently, the evolution of output. These fluctuations do not rest on any exogenous shocks. Instead, both the impulse source and the propagation mechanism lie with the structural characteristics of the economic environment. In particular, the occupational choice is the source of non-monotonicities that generate fluctuations and propagate them into fluctuations of output growth.

5 Conclusion

In this paper, our endeavour was to contribute to the emerging body of literature that studies the dynamic behaviour of endogenous market structures in dynamic general equilibrium models. We showed that an overlapping generations setting, combined with the idea that entry decisions are made through an occupational choice process, can lead to potentially interesting implications concerning these dynamic patterns. We showed that the intrinsic dynamics of the industry can lead to fluctuations, either though damped oscillations or limit cycles. These results represent yet another example on how endogenous forces can cause fluctuations in economic dynamics.

A note of caution merits discussion here, given the fact that that our paper's dynamics are characterised by periodic orbits that may resemble the type of fluctuations we observe in the data. We believe that a better interpretation of our results should entail a correspondence to low frequency waves in industry activity, such as those presented by Comin and Gertler (2006) for example, rather than the high frequency fluctuations that are more suitably attributed to the occurrence of short-term business cycles. For this reason, we need to clarify that our analysis in under no circumstances an attempt to invalidate other explanations for the cyclicality of economic dynamics, based on the idea of exogenous shocks – explanations that we actually view as being indubitably important. The main message form our work is that the cyclical behaviour of economies, in addition to being a response to changing economic conditions, may also reflect characteristics that render them inherently volatile. As we indicated at the very beginning of this paper, other authors have asserted the same through their research work, thus offering some momentum to this idea.

The model we presented is simple enough to guarantee a clear understanding of the mechanisms that are involved in the emergence of the basic results, without blurring either their transparency or their intuition. Of course, there is certainly a large scope for getting additional implications by modifying or enriching some of the model's founding characteristics. One obvious direction is to assume that the oligopolistic industry supplies firms with different varieties of capital goods while, at the same time, retaining the important characteristic of endogenous occupational choice. The ensuing process of capital accumulation could set in motion some very interesting implications concerning economic dynamics. We believe that this set up should certainly offer a potentially fruitful avenue for future research work.

Appendix

Proofs of Lemmas 1-4

The Jacobian matrix associated with the planar system in (33)-(34) is

$$\begin{pmatrix} F_{Z_t}(\hat{Z},\hat{N}) & F_{N_t}(\hat{Z},\hat{N}) \\ H_{Z_t}(\hat{Z},\hat{N}) & H_{N_t}(\hat{Z},\hat{N}) \end{pmatrix},$$

where $\hat{N}=\hat{Z}$ is given in (31). Some straightforward algebra with the use of Equations (31), (33) and (34), reveals that the trace (T) and the determinant (D) are equal to

$$T = F_{Z_t}(\hat{Z},\hat{N}) + H_{N_t}(\hat{Z},\hat{N}) = -\delta\left\{\theta - \frac{a(1+\delta\theta)}{(1-a)[n-(1+\delta)]}\right\}, \tag{A1}$$

and

$$D = F_{Z_t}(\hat{Z},\hat{N})H_{N_t}(\hat{Z},\hat{N}) - F_{N_t}(\hat{Z},\hat{N})H_{Z_t}(\hat{Z},\hat{N}) = \frac{\delta a(1+\delta\theta)}{(1-a)[n-(1+\delta)]}, \tag{A2}$$

respectively. Furthermore, the eigenvalues are the roots of the polynomial $\lambda^2 - T\lambda + D$, i.e.,

$$\lambda_1 = \frac{T-\sqrt{T^2-4D}}{2} \text{ and } \lambda_2 = \frac{T+\sqrt{T^2-4D}}{2}. \tag{A3}$$

To ensure the stability of the steady state, we want the eigenvalues to satisfy $|\lambda_1| < 1$ and $|\lambda_2| < 1$. Given that $\lambda_1 + \lambda_2 = T$ and $\lambda_1\lambda_2 = D$, two necessary but not sufficient conditions for stability are $-1 < D < 1$ and $-2 < T < 2$. Evidently, the determinant is positive by virtue of (A2); therefore we can use (A2) to find that $D < 1$ corresponds to the restriction

$$n - 1 > \delta + \frac{\delta a(1+\delta\theta)}{1-a}. \tag{A4}$$

Furthermore, note that we can use (A1) and (A2) to get

$$T = D - \delta\theta. \tag{A5}$$

As we constrain ourselves to $D < 1$, Equation (A5) reveals that $T < 2$. Therefore, we want to obtain a restriction for which $T > -2$. Using (A5), it can be very easily established that a sufficient (but not necessary) condition for this is given by

$$\delta\theta < 2. \tag{A6}$$

In addition to the above, we will rule out complex eigenvalues by imposing the parameter restriction that ensures $T^2 - 4D \geq 0$. Specifically,

$$T^2 \geq 4D \Leftrightarrow$$

$$(D-\delta\theta)^2 \geq 4D \Leftrightarrow$$

$$D^2 - 2(2+\delta\theta)D + (\delta\theta)^2 \geq 0 \Leftrightarrow$$

$$\Phi(D) \geq 0. \tag{A7}$$

It is $\Phi(0) = (\delta\theta)^2 > 0$ and $\Phi(1) = 1 - 2(2+\delta\theta) + (\delta\theta)^2 < 0$ by virtue of (A6). Furthermore, $\Phi' = 2D - 2(2+\delta\theta) < 0$ for $D \in (0,1)$. Hence, for $T^2 - 4D \geq 0$ to hold we need the restriction $D < \tilde{D}_{\min}$, where $\tilde{D}_{\min}$ is the lowest-valued root of $\Phi(\tilde{D}) = 0$. We can then use (A2) to establish that

$$D < \tilde{D}_{\min} \Leftrightarrow,$$

$$D < 2 + \delta\theta - 2\sqrt{1+\delta\theta} \Leftrightarrow.$$

$$n-1 \geq \delta+\frac{\delta a(1+\delta\theta)}{(1-a)(2+\delta\theta-2\sqrt{1+\delta\theta})} \equiv \Omega(\delta)\,. \tag{A8}$$

However, notice that $2+\delta\theta-2\sqrt{1+\delta\theta} \in (0,1)$ by virtue of (A6). This implies that the restriction in (A8) ensures that the condition in (A4) is also satisfied.

Now, check that $\delta\theta > 2+\delta\theta-2\sqrt{1+\delta\theta}$. By virtue of (A8), this means that

$$n \geq 1+\delta+\frac{\delta a(1+\delta\theta)}{(1-a)\delta\theta}\,. \tag{A9}$$

Consequently, combining (A9) and (A5), we can establish that the trace T is negative, i.e., $T \in (-2,0)$ which, combined with (A3), reveals that both eigenvalues λ_1 and λ_2 are negative. It can be easily established that $\lambda_2 > -1$, whereas $\lambda_1 > -1$ holds as long as

$$\frac{T-\sqrt{T^2-4D}}{2} > -1 \Leftrightarrow$$

$$2+T > \sqrt{T^2-4D}\,.$$

Given $T > -2$, we can use the above expression to get

$$(2+T)^2 > (\sqrt{T^2-4D})^2 \Leftrightarrow$$

$$4+4T > -4D \Leftrightarrow$$

$$D+T+1 > 0 \Leftrightarrow$$

$$D > \frac{\delta\theta-1}{2}\,. \tag{A10}$$

which holds unambiguously when $\delta\theta \leq 1$. Hence, in this case $0 > \lambda_2 > \lambda_1 > -1$ holds – a result ensuring that there is convergence to the long-run equilibrium and that it is oscillatory (or cyclical).

Now consider the case where $\delta\theta \in (1,2]$. The condition in (A10) can be written as

$$n-1 < \Xi(\delta)\,, \tag{A11}$$

where

$$\Xi(\delta) \equiv \delta+\frac{\delta a(1+\delta\theta)}{1-a}\frac{2}{\delta\theta-1}\,. \tag{A12}$$

Recalling that we are considering values for which $\delta\theta \in (1,2]$, we can determine that $\lim_{\delta\to(1/\theta)_+}\Xi(\delta) = +\infty$ and $\Xi\left(\frac{2}{\theta}\right) = 2\left(1+\frac{3a}{1-a}\right) < n-1$ by assumption. As long as $\Xi(\delta)$ cuts the $n-1$ line only once, then there is $\tilde{\delta}$ such that $\Xi(\tilde{\delta}) = n-1$. Furthermore, note that for $\delta\theta \in (1,2]$ we have $\frac{1}{2+\delta\theta - 2\sqrt{1+\delta\theta}} < \frac{2}{\delta\theta - 1}$. Given (A8), (A12) and the assumption $n > 1 + 2\left(1 + \frac{3a}{1-a}\right)$, this implies that $\Xi(\delta) > \Omega(\delta) \quad \forall \delta \le 2/\theta$, i.e., the condition in (A8) always holds given our assumptions.

The previous analysis implies that (A11) holds when $\delta < \tilde{\delta}$ and therefore $0 > \lambda_2 > \lambda_1 > -1$. The steady state $\hat{N}$ is locally stable. However, when $\delta > \tilde{\delta}$ we have $0 > \lambda_2 > -1 > \lambda_1$ and the steady state $\hat{N}$ is a saddle point. Evidently, at $\delta = \tilde{\delta}$ we have $\lambda_1 = -1$. Combined with $\lambda_2 \in (-1, 0)$, we can use Theorem 8.4 in Azariadis (1993) to deduce that the dynamical system undergoes a flip (or period doubling) bifurcation so that there exists a cycle of at least two periods.

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